ON THE CHARACTERISTIC DRYING CURVE

R. B. KEEY

University of Canterbury, Christchurch, New Zealand

and

M. SUZUKI*

Tohoku University, Sendai, Japan

Abstract—The theoretical foundation for the concept of the characteristic drying curve is examined by considering the drying out of a porous, non-hygroscopic slab of infinite extent. Under intensive drying conditions when a drying front appears, the characteristic drying curve is a function of the relative intensity of drying, whereas under less intensive drying conditions it is not. In the limit of the slow drying of thick, fairly impervious materials, a single linear characteristic drying curve appears.

NOMENCLATURE

a, interfacial area per unit volume $[\mathrm{m}^{-1}]$;
a, a', coefficients;
b, material thickness $[\mathrm{m}]$;
D, relative molar mass, moisture vapour/air;
f, relative evaporation rate, N_A/N_A^0;
f_e, relative evaporation rate when moisture front reaches the base of the material;
F, mass-transfer coefficient, equation (1) $[\mathrm{mol\ m}^{-2}\mathrm{s}^{-1}]$;
h, external heat-transfer coefficient $[\mathrm{W\,m}^{-2}\mathrm{K}^{-1}]$;
K, overall mass-transfer coefficient, equation (5) $[\mathrm{kg\,m}^{-2}\mathrm{s}^{-1}]$;
K_Y, mass-transfer coefficient, based upon humidity potential, equation (3) $[\mathrm{kg\,m}^{-2}\mathrm{s}^{-1}]$;
m, hygrothermal gradient $[\mathrm{K}^{-1}]$;
m_{EW}, $(Y_E - Y_W)/(T_E - T_W)\ [\mathrm{K}^{-1}]$;
m_{SG}, $(Y_S - Y_G)/(T_S - T_G)\ [\mathrm{K}^{-1}]$;
m_{WG}, $(Y_W - Y_G)/(T_W - T_G)\ [\mathrm{K}^{-1}]$;
M_A, molar mass of moisture vapour $[\mathrm{kg\,mol}^{-1}]$;
N_A, evaporation rate per unit exposed surface (flux) $[\mathrm{kg\,m}^{-2}\mathrm{s}^{-1}]$;
T_E, temperature of evaporative interface $[\mathrm{K}]$;
T_G, dry-bulb temperature $[\mathrm{K}]$;
T_S, surface temperature $[\mathrm{K}]$;
T_W, wet-bulb temperature $[\mathrm{K}]$;
U, overall heat-transfer coefficient $[\mathrm{W\,m}^{-2}\mathrm{K}^{-1}]$;
X, moisture content, dry basis;
$\bar{X}$, mean moisture content;

X_{cr}, critical moisture content;
X_0, initial moisture content;
X_1, moisture content at the base of the material $(\eta = 1)$;
X^*, equilibrium-moisture content;
y_G, moisture mole fraction in bulk air;
y_S, moisture mole fraction in air adjacent to surface;
Y_E, saturation humidity at evaporative-interface temperature;
Y_G, humidity of bulk air;
Y_S, humidity of air adjacent to surface;
Y_W, saturation humidity at wet-bulb temperature;
$\mathscr{D}$, apparent diffusion coefficient of moisture vapour through dried-out material $[\mathrm{m}^2\mathrm{s}^{-1}]$;
$\mathscr{D}_a$, apparent diffusion coefficient for moisture movement through wet material $[\mathrm{m}^2\mathrm{s}^{-1}]$;
$\mathscr{D}_{AB}$, diffusion coefficient of moisture vapour through air $[\mathrm{m}^2\mathrm{s}^{-1}]$;
$\mathscr{N}$, relative intensity of drying, equation (28) $[\mathrm{m}^2\mathrm{s}^{-1}]$;
$\mathscr{N}_0$, relative intensity of drying based on the limiting value of $\mathscr{D}_a$ at low moisture contents.

Greek symbols
α, function of ζ, equation (19);
β, dimensionless transport property of material, equation (16);
γ, evaporative-resistance coefficient, equation (26);
δ, fractional penetration of drying front;
ΔH_v, latent heat of vaporization $[\mathrm{J\,kg}^{-1}]$;
ζ, number of transfer units;

*On leave at the University of Canterbury, Christchurch, New Zealand.

η,   relative depth;

λ,   apparent thermal conductivity of material $[Wm^{-1}K^{-1}]$;

μ_D,   diffusion-resistance coefficient;

ρ_s,   bulk density of dried-out solids $[kg\,m^{-3}]$;

φ,   humidity–potential correction factor, equation (2);

Φ,   characteristic moisture content, equation 4;

ξ,   fractional depth of recession of evaporative interface.

INTRODUCTION

THE RIGOROUS process design of drying plant is still an unsolved problem. A prospective purchaser of a dryer thus relies on the experience of the chemical-plant manufacturer on the behaviour of his plant and, where a new product is to be handled, extensive tests at bench and pilot scale are usually required. Reliable scale-up procedures are not available [1], and the interpretation of pilot-scale data forms part of the art of the chemical-plant manufacturer.

One promising process-design method [2] is based on the postulate that it is possible to draw up a characteristic drying curve for a specific material from bench-scale drying data. The relative ease of drying is assumed to be a unique function of a "characteristic" moisture content so that a lumped-parameter analysis of drying behaviour becomes possible.

The evaporation flux from a fully wetted surface is described in many texts, e.g. Treybal [3]. Equation (3.1) of this author becomes

$$N_A^0 = FM_A \ln\left(\frac{1-y_G}{1-y_S}\right) \tag{1}$$

for the flux between positions 1 and 2 in Fig. 1. In equation (1), F is a mass-transfer coefficient [4], y_G is the mole fraction of moisture vapour in the bulk
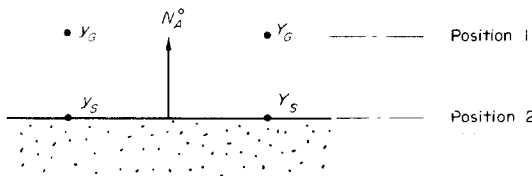

FIG. 1. Evaporation from a moist surface.

airstream and y_S is the mole fraction of moisture vapour adjacent to the moist surface. It is more convenient for bookkeeping to describe the flux in terms of the corresponding humidity potential $(Y_S - Y_G)$. The result of the algebraic manipulation of equation (1), as shown by Ashworth and Keey [5], is the following expression

$$N_A^0 = M_A\left(\frac{F}{D}\right)\psi(Y_S - Y_G) \tag{2}$$

where

$$\psi = \frac{D}{D+Y_S} \frac{(D+Y_S)\ln\dfrac{(D+Y_S)}{(D+Y_G)}}{(Y_S - Y_G)}.$$

The ratio $M_A F/D$ is a new mass-transfer coefficient, K_Y, defined in terms of the humidity potential $(Y_S - Y_G)$, where D is the ratio of the molar mass of the moisture vapour to that of the dry gas. The parameter ψ represents the "correction" to this linear potential $(Y_S - Y_G)$ due to the inappropriateness of its choice and the "correction" due to high mass-transfer effects which is often negligible under most drying conditions. The humidity–potential correction factor is then evaluated simply as $D/(D + Y_S)$.

Whenever the surface is fully wetted, the temperature there reaches the wet-bulb temperature T_W, and Y_S can be evaluated as Y_W, the saturation humidity at the wet-bulb temperature.

It is postulated that the moisture efflux from a drying solid can be described by a similar expression to equation (2), namely

$$N_A = fK_Y\psi(Y_W - Y_G) \tag{3}$$

in which the newly introduced parameter f takes on the following values:

unhindered evaporation $f = 1$; $\Phi \geqslant 1$;
hindered evaporation $0 < f < 1$; $0 < \Phi < 1$.

The function Φ is the so-called characteristic moisture content, and is defined in terms of the critical point, the smallest mean moisture content of the sample for the evaporation to remain ostensibly hindered by the presence of the solid. Generally, Φ is calculated as a relative free moisture content, i.e.

$$\Phi = (X - X^*)/(X_{cr} - X^*) \tag{4}$$

where X is the moisture content (dry basis), X^* the equilibrium-moisture content and X_{cr} the critical moisture content. It is assumed that a unique relationship between f and Φ can be found for a specific material.

The object of this paper is to examine the theoretical foundation (if any) of the above concept. We shall consider the drying out of a non-hygroscopic ($X^* = 0$), porous slab of infinite extent. These restraints, while providing a simplification of the analysis, are considered not to restrict unduly the conclusions drawn therefrom.

To avoid specifying a particular mode of moisture migration through the solid, we shall take initially a lumped-parameter model of the drying process. Subsequently it will be necessary to consider moisture-content profiles in the solid. Then it will be assumed that moisture movement is Fickian.

There have been two principal pictures to describe the falling off in moisture loss as a body dries out: one in terms of the fraction of the exposed surface that is wetted, the other in terms of an evaporative plane that withdraws into the body. The wetted-surface model gives rise to drying curves which are concave downwards [6]. However, these curves are untypical of the drying of porous, non-hygroscopic materials [7], although they can approximate those for hygroscopic materials such as timber [8]. On the other hand, the receding-plane model is supported by experimental data on the drying of beds of glass beads soaked with a range of liquids [9]. For these reasons we shall adopt the latter model.

THEORY

1. The temperature of the evaporative interface

Figure 2 shows the temperature and humidity profiles through the air and dried-out zone when the evaporative plane has withdrawn to a relative depth ξ.
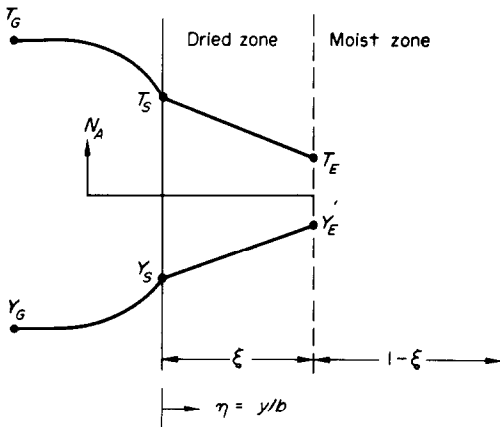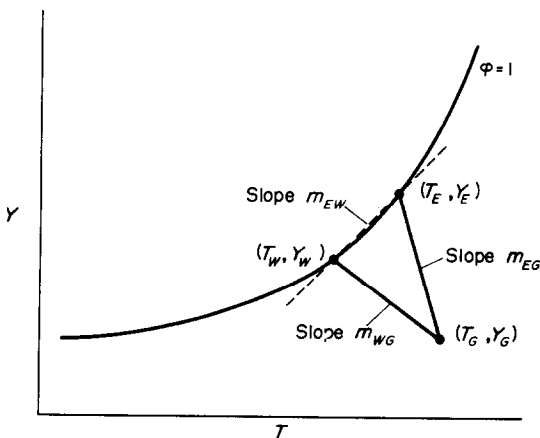

FIG. 2. Temperature and humidity profiles.

FIG. 3. Hygrothermal gradients.

(body thickness $= 1$). Under these conditions, the moisture flux can be evaluated by two expressions:

1. Transfer through the air boundary layer

$$N_A = K_Y \psi_S (Y_S - Y_G) \qquad (5)$$

in which $\psi_S = D/(D + Y_S)$ under most drying conditions.

2. Transfer through the dried-out zone

$$N_A = \frac{\mathscr{D}_{AB}}{\mu_D \xi bD} \cdot \psi_E (Y_E - Y_S) \qquad (6)$$

for which $\mathscr{D}_{AB}$ is the moisture–vapour diffusivity, μ_D is the so-called diffusion resistance coefficient, b is the body thickness, and ψ_E is the humidity–potential correction factor approximated by $D/(D + Y_E)$.

In equation (6), the effective K-type mass-transfer coefficient is $\mathscr{D}_{AB}/\mu_D \xi bD$.

Now, under most commonly occurring drying conditions,

$$Y_S < Y_E \ll D = 0.6228 \text{ (air–steam system)}$$

with

$$(Y_E - Y_G) < Y_E$$

so that with little error one may write

$$\psi \sim \psi_S \sim \psi_E. \qquad (7)$$

It is convenient to define an overall K-type mass-transfer coefficient K:

$$N_A = K\psi (Y_E - Y_G). \qquad (8)$$

Combination of equations (5), (6) and (8) with equation (7) yields an expression for the overall mass-transfer resistance $1/K$:

$$\frac{1}{K} = \frac{\mu_D \xi bD}{\mathscr{D}_{AB}} + \frac{1}{K_Y} \qquad (9)$$

or, for convenience,

$$\frac{1}{K} = \frac{\xi b}{\mathscr{D}} + \frac{1}{K_Y} \qquad (10)$$

in which $\mathscr{D}$ is an apparent diffusion coefficient ($\mathscr{D} = \mathscr{D}_{AB}/\mu_D D$).

A similar identity may be written for the overall heat-transfer coefficient U in terms of the external heat-transfer coefficient h and the effective thermal conductivity λ of the dry porous mass:

$$\frac{1}{U} = \frac{\xi b}{\lambda} + \frac{1}{h}. \qquad (11)$$

Now the heat received by the body results in evaporation of moisture and the slow rise of the body temperature level which is reflected in the rate of rise

of the interfacial temperature $dT_E/d\tau$. The heat- and mass-transfer fluxes are thus coupled:

$$U(T_G - T_E) = K\psi(Y_E - Y_G)\Delta H_{VE} + \left[\frac{s\rho_s}{a}\right]\frac{dT_E}{d\tau} \quad (12)$$

in which ΔH_{VE} is the latent heat of vaporization at temperature T_E (the heat of desorption is zero for a non-hygroscopic body), s is the mean heat capacity of the body per unit bone-dry mass, a is the external surface of the body per unit volume, and ρ_s is the bulk density of the bone-dry material. The sensible-heat term has a magnitude about fiftyfold smaller than the latent-heat term, when the moisture is water, and thus may be neglected. The mean hygrothermal gradient $(\Delta Y/\Delta T)$ thus becomes:

$$m_{EG} = \frac{Y_E - Y_G}{T_E - T_G} = \frac{-U}{K\psi\,\Delta H_{VE}}. \quad (13)$$

On substituting the values of K and U from equations (10) and (11) respectively, we find

$$m_{EG} = -\frac{1 + K_Y\xi b/\mathscr{D}}{1 + h\xi b/\lambda}\cdot\frac{h}{K_Y}\cdot\frac{1}{\psi\,\Delta H_{VE}}. \quad (14)$$

Equation (14) can be conveniently rewritten as

$$m_{EG} = -\frac{1 + \xi Bi}{1 + \beta\xi Bi}\cdot\frac{h}{K_Y}\cdot\frac{1}{\psi\,\Delta H_{VE}} \quad (15)$$

in which Bi is the mass-transfer Biot number $(Bi = K_Y b/\mathscr{D})$. The coefficient is given by

$$\beta = \frac{h\mathscr{D}}{K_Y\lambda} = \frac{Nu}{Sh}\cdot\frac{Lu_S}{Lu_G} \quad (16)$$

where Lu_S is the Luikov number for transport through the dried-out zone, and Lu_G for transport through the air boundary layer. If the moisture is aqueous and the dry gas air, $Nu \sim Sh$ and $Lu_G \sim 1$ so that

$$\beta \sim Lu_S. \quad (17)$$

The coefficient β is essentially the Luikov number for transport through the dried-out zone. Normally this coefficient is less than unity, and for beds of non-hygroscopic particles values of β between 0·06 and 0·1 are recorded [14]. When $\xi = 0$, $T_E = T_S = T_W$ (the wet-bulb temperature) and equation (15) reduces to

$$m_{EG} = m_{SG} = m_{WG} = \frac{Y_W - Y_G}{Y_W - T_G} = -\frac{h}{K}\cdot\frac{1}{\psi}\frac{1}{\Delta H_{VW}} \quad (18)$$

for the hygrothermal gradient. If the variation of latent heat of vaporization with temperature is assumed negligible over the temperature range involved, we find from equations (15) and (18) that

$$\frac{m_{EG}}{m_{WG}} = \frac{1 + \xi Bi}{1 + \beta\xi Bi} = \alpha(\xi). \quad (19)$$

If the drying is slow enough for thermodynamic equilibrium to prevail at the evaporative interface, then we find from geometrical considerations:

$$(Y_E - Y_G) = (Y_E - Y_W) + (Y_W - Y_G) \quad (20)$$

and

$$m_{EG}(T_E - T_G) = m_{EW}\{(T_E - T_G) - (T_W - T_G)\} \\ m_{WG}(T_W - T_G). \quad (21)$$

By algebraic manipulation, we get

$$\frac{T_G - T_E}{T_G - T_W} = \frac{m_{EW} - m_{WG}}{m_{EW} - m_{EG}}. \quad (22)$$

Comparison of equations (19) and (22) yields

$$\frac{T_G - T_E}{T_G - T_W} = \frac{1 - m_{WG}/m_{EW}}{1 - \alpha m_{WG}/m_{EW}}. \quad (23)$$

Equation (23) gives us an expression for T_E in terms of the depth of the evaporative interface $\xi(\alpha)$ and the hygrothermal ratio m_{WG}/m_{EW}. This latter parameter is the ratio of the mean slope of the wet-bulb depression curve to the mean slope of moisture–saturation curve over the temperature range involved. Although the hygrothermal ratio is essentially independent of wet-bulb depression, the curves being almost linear, the ratio is a strong function of wet-bulb temperature itself, as Fig. 4 shows. The coefficient $\alpha(\xi)$ approaches
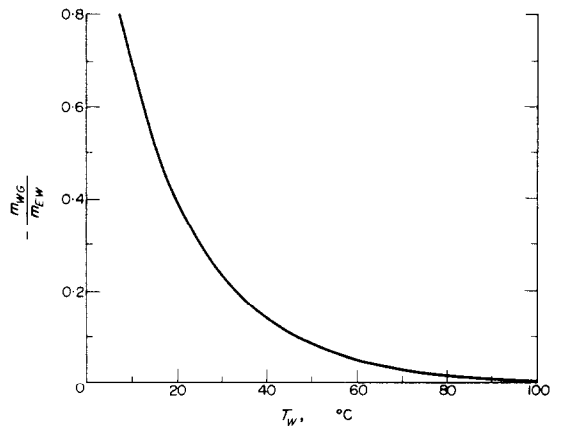
FIG. 4. Hygrothermal ratio $-m_{WG}/m_{EW}$ for air–stream system. Range of wet-bulb depressions 2–100°C.

a limiting value for high Bi, which is $1/\beta$ from equation (19). Under these conditions, T_E approaches an asymptotic temperature that has been called the pseudo-wet-bulb temperature [10], and has been experimentally observed in the drying of beds of non-hygroscopic material such as powdered chalk [11].

2. The relative evaporation rate

The relative evaporation rate f may be determined from the heat-transfer flux to the evaporative interface:

$$f = \frac{U(T_G - T_E)}{h(T_G - T_W)}. \tag{24}$$

The overall heat-transfer coefficient U is described by equation (11) and the temperature ratio by equation (23). These substitutions yield

$$f = \frac{1}{1 + h\xi b/\lambda} \frac{1 - m_{WG}/m_{EW}}{[1 - \alpha m_{WG}/m_{EW}]}. \tag{25}$$

Now, we have $\beta Bi = hb/\lambda$ and $\alpha = (1 + \xi Bi)/(1 + \beta\xi Bi)$, so equation (25) can be rewritten as

$$f = 1 \Big/ \left[1 + \frac{(\beta - m_{WG}/m_{EW})}{(1 - m_{WG}/m_{EW})}\right]\xi Bi. \tag{26}$$

It is convenient to lump the term $(\beta - m_{WG}/m_{EW})/(1 - m_{WG}/m_{EW})$ into a single hygrothermal parameter γ. Values of γ depend upon the hygrothermal ratio m_{WG}/m_{EW} and on the properties of the dried solids (β), although at very high wet-bulb temperatures, as Fig. 5 shows, the solid properties are the more determinative.
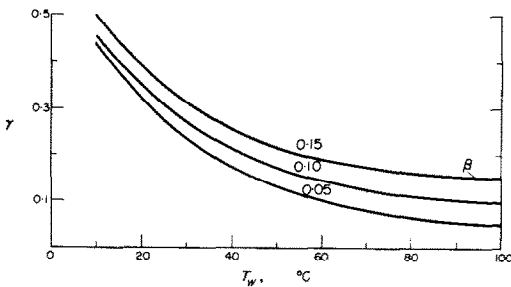
FIG. 5. The evaporative-resistance coefficient γ for solids wet with water and drying in air.

This parameter γ, which we shall call the evaporative resistance coefficient, approaches β itself as the wet-bulb temperature approaches the boiling point of the moisture. We thus write
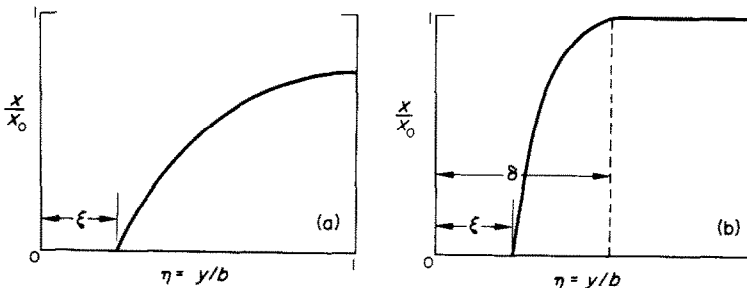
$$f = \frac{1}{1 + \gamma\xi Bi} \tag{27}$$

which describes the attenuation of moisture loss as ξ increases. The coefficient γ is a complex thermophysical function, but at constant wet-bulb temperature γ is a constant for a specific material. Further, if we are concerned with a situation where Bi is a constant, then the relative drying rate f is solely a function of ξ, the depth of the evaporative plane. These are the presumptive grounds for the concept of the characteristic drying curve.

3. Intensity-of-drying regimes

Two kinds of moisture-content profiles may appear. If the intensity of drying is severe enough, a drying-zone wave of extent $\delta(\delta < 1)$ will sweep through the material (Fig. 6(b)). Under these conditions, the moisture content at the exposed surface can be zero while the base material is still at its initial moisture content. If the intensity were milder, then $X/X_{0|\eta=1}$ will always be less than unity before the critical point appears (Fig. 6(a)).

It is shown in the appendix that high-intensity drying in the above terms occurs whenever

$$\mathcal{N} = N_A^0 b/\rho_S X_0 \mathcal{D}_a > 2 \tag{28}$$

in which N_A^0 is the initial (unhindered) evaporation flux, X_0 is the initial moisture content (dry basis), and $\mathcal{D}_a$ is the apparent diffusivity, assumed constant, under a gradient of concentration $\rho_S X$. A similar criterion can be obtained for the variable-diffusivity case. Substitution into inequality (28) suggests that under mild cross-circulation conditions the low-intensity profile, Fig. 6(a), pertains, while under more rigorous through-circulation of the material, the high-intensity profile can occur. Further, a drying front will be observed even under very mild conditions whenever the bed is deep enough. This situation is seen in the drying of food grains in silos [13].

4. Low-intensity drying

If the moisture movement through the solid is describable in terms of a constant "diffusion coefficient" $\mathcal{D}_a$, then parabolic moisture-content profiles sweep

FIG. 6. Possible moisture-content profiles: (a) slow drying; (b) fast drying.

through the material. Such profiles have been recorded in the drying of non-hygroscopic materials, such as glass beads and powdered chalk (11). Accordingly, we have

$$X = X_1 + a(1 - \eta)^2, \quad 1 > \eta > \xi \tag{29}$$

for the profiles. The coefficient a can be determined from the known evaporation flux at a given depth ξ, equation (27).

$$N_A = f N_A^0 = N_A^0/(1 + \gamma \xi Bi). \tag{30}$$

But also we have,

$$N_A = \frac{\rho_S \mathscr{D}_a}{b} \frac{\partial X}{\partial \eta}\bigg|_{\eta = \xi} \tag{31}$$

so that we find, on equating expressions (30) and (31):

$$\frac{\partial X}{\partial \eta}\bigg|_{\eta = \xi} = \frac{N_A^0 b}{\rho_S \mathscr{D}_a (1 + \gamma \xi Bi)}. \tag{32}$$

Differentiation of equation (29) yields

$$\frac{\partial X}{\partial \eta}\bigg|_{\eta = \xi} = -2a(1 - \xi) \tag{33}$$

so that the unknown coefficient becomes

$$a = -N_A^0 b/2\rho_S \mathscr{D}_a(1 + \gamma \xi Bi)(1 - \xi). \tag{34}$$

The moisture content varies from zero (between the exposed surface to a depth ξ) to a value X_1 at the base of the material. This moisture content can be found by re-arranging equation (29); the result is

$$x_1 = N_A^0 b(1 - \xi)/2\rho_S \mathscr{D}_a(1 + \gamma \xi Bi). \tag{35}$$

The mean moisture content over the whole material is given by

$$\frac{\overline{X}}{X_0} = \frac{\int_\xi^1 X \, d\eta}{X_0}. \tag{36}$$

On expanding X as a function of η, we obtain

$$\frac{\overline{X}}{X_0} = \left\{ [X_1 \eta]_\xi^1 - \left[\frac{a}{3}(1 - \eta)^3 \right]_\xi^1 \right\} \bigg/ X_0 \tag{37}$$

$$= \frac{2}{3} \frac{X_1}{X_0} (1 - \xi). \tag{38}$$

Now, the characteristic moisture content is defined as

$$\Phi = \frac{\overline{X}/X_0}{X_{cr}/X_0}. \tag{39}$$

Comparison of equations (38) and (39) yields

$$\Phi = \frac{X_1(1 - \xi)}{X_{1|cr}} \tag{40}$$

where $X_{1|cr}$ is the moisture content at the base of the material at the critical point. This moisture content

can be determined from equation (35) by putting $\xi = 0$; we find

$$X_{1|cr} = N_A^0 b/2\rho_S \mathscr{D}_a. \tag{41}$$

Combination of equations (35), (40) and (41) gives

$$\Phi = \left(\frac{1}{1 + \gamma \xi Bi} \right)(1 - \xi)^2. \tag{42}$$

Now, the relative evaporation rate is given by

$$f = 1/(1 + \gamma \xi Bi) \tag{27}$$

which, on re-arrangement, gives us an expression for ξ:

$$\xi = (1 - f)/f\gamma Bi. \tag{43}$$

Substitution of equation (43) into equation (42) leads to a non-explicit expression for the relative drying rate f in terms of the characteristic moisture content Φ:

$$\Phi = f\{1 - (1 - f)/f\gamma Bi\}^2. \tag{44}$$

Except close to the end of drying, equation (44) predicts a linear falling-rate curve $f = \Phi$ when γBi is large, i.e. for thick, fairly impervious materials. The predicted terminal drying rate is

$$f_e = 1/(1 + \gamma Bi). \tag{45}$$

This result also follows from equation (27) when $\xi = 1$, i.e. when the material is completely dry. Thus equation (44) is subject to the restraint that $f \geqslant f_e$.

5. High-intensity drying

In this case, the moisture-content profile exists over the region, $\xi < \eta < \delta(\delta < 1)$. Instead of equation (29), the following expression then holds for the profile:

$$X = X_0 + a'(\delta - \eta)^2 \tag{46}$$

where a' is a coefficient to be determined. Equation (32),

$$\frac{\partial X}{\partial \eta}\bigg|_{\eta = \xi} = N_A^0 b/\rho_S \mathscr{D}_a(1 + \gamma \xi Bi) \tag{32}$$

yields the moisture-content gradient at the evaporative interface. Differentiation of equation (46) gives, with equation (32),

$$N_A^0 b/\rho_S \mathscr{D}_a(1 + \gamma \xi Bi) = -2a'(\delta - \xi) \tag{47}$$

which gives us the value of a'.

The mean moisture content in the material is given by

$$\frac{\overline{X}}{X_0} = \frac{\int_\xi^\delta X \, d\eta + \int_\delta^1 X_0 \, d\eta}{X_0} \tag{48}$$

$$= \frac{\int_\xi^\delta (X - X_0) \, d\eta + \int_\xi^1 X_0 \, d\eta}{X_0}. \tag{49}$$

Whence we find

$$\bar{X}/X_0 = 1 - \xi - (\delta - \xi)/3 . \quad (50)$$

Equations (46) and (47) yield us a relationship for the extent of the drying front $(\delta - \xi)$:

$$\delta - \xi = \frac{a'(\delta - \xi)^2}{a'(\delta - \xi)} = \frac{-X_0}{-N_A^0 b/2\rho_S \mathscr{D}_a(1 + \gamma\xi Bi)} . \quad (51)$$

Equation (51) can be used to eliminate δ from the profile [equation (50)]:

$$\bar{X}/X_0 = 1 - \xi - 2\rho_S \mathscr{D}_a(1 + \gamma\xi Bi)X_0/3N_A^0 b . \quad (52)$$

At the critical point, $\xi = 0$, so the critical moisture content becomes:

$$X_{cr}/X_0 = 1 - 2\rho_S \mathscr{D}_a X_0/3N_A^0 b . \quad (53)$$

The characteristic moisture content follows:

$$\Phi = \frac{\bar{X}/X_0}{X_{cr}/X_0} = \frac{1 - \xi - 2\rho_S \mathscr{D}_a(1 + \gamma\xi Bi)X_0/3N_A^0 b}{1 - 2\rho_S \mathscr{D}_a X_0/3N_A^0 b} . \quad (54)$$

Since $N_A^0 b/\rho_S \mathscr{D}_a X_0$ is the dimensionless intensity of drying $\mathscr{N}$ one has

$$\Phi = \frac{1 - \xi - 2(1 + \gamma\xi Bi)/3\mathscr{N}}{1 - 2/3\mathscr{N}} . \quad (55)$$

The depth of the recessed evaporation plane ξ is given by equation (43); substitution of this value in the above expression yields a non-explicit expression for f in terms of Φ:

$$\Phi = \frac{1 - (1-f)/f\gamma Bi - 2/3\mathscr{N}f}{1 - 2/3\mathscr{N}} . \quad (56)$$

In this case, we see that there is no unique characteristic curve independent of rate, except for very high $\mathscr{N}$ when

$$\Phi = 1 - (1-f)/f\gamma Bi . \quad (57)$$

Inspection of equation (55) shows that this high-intensity limit also corresponds to

$$\Phi = 1 - \xi . \quad (58)$$

The moisture content is merely the sodden fraction of the body. A similar situation exists in the early stages of low-intensity drying when $X_1 \sim X_{1|cr}$ [equation (40)].

When the drying front has swept through the material, $\delta > 1$ and equation (56) no longer applies. In the first stage of drying, the front will have reached a position given by

$$\delta - \xi = 2\rho_S \mathscr{D}_a(1 + \gamma\xi Bi)X_0/N_A b$$
$$= 2(1 + \gamma\xi Bi)/\mathscr{N}f . \quad (51)$$

For equation (56) to be valid,

$$\delta = \xi + 2(1 + \gamma\xi Bi)/\mathscr{N}f < 1 . \quad (59)$$

The value of ξ is given by equation (43), so inequality (59) reduces to

$$f > \frac{(1/\gamma Bi + 2/\mathscr{N})}{(1 + 1/\gamma Bi)} \quad (60)$$

after some manipulation. This expression has the limiting value of

$$f > 1/(1 + \gamma Bi) = f_e \quad (61)$$

at high values of $\mathscr{N}$ when the front is negligibly thin.

In the second stage of drying, a moisture-content profile, typical of low-intensity drying (Fig. 6(a)), will be formed. The moisture content at the base of the material falls to a value given by equation (35):

$$X_1 = N_A^0 b(1 - \xi)/2\rho_S \mathscr{D}_a(1 + \gamma Bi) \quad (35)$$
$$= \mathscr{N}X_0(1 - \xi)f/2 . \quad (62)$$

The mean moisture content is found from equation (38) which can be written in the following form:

$$\bar{X}/X_0 = 2\mathscr{N}f(1 - \xi)2/3 \quad (63)$$

by using the above value for X_1. The critical moisture content is given by equation (53) for the value under high-intensity drying:

$$X_{cr}/X_0 = 1 - 2/3\mathscr{N} . \quad (53)$$

Equations (53) and (63) yield the characteristic moisture content, and on noting that $\xi = (1 - f)/f\gamma Bi$ [equation (43)], we obtain:

$$\Phi = \frac{\bar{X}/X_0}{X_{cr}/X_0} = \mathscr{N}f(1 - (1-f)/f\gamma Bi)^2/3(1 - 2/3\mathscr{N}) . \quad (64)$$

6. Variable diffusivity

Normally, the apparent diffusion coefficient is a function of concentration level, often becoming smaller as the moisture content is reduced. Then the moisture-content gradient close to the evaporative interface will be greater than that in the constant-diffusivity case for a comparable drying rate. The moisture content profile is no longer parabolic. To simulate this situation, we take a cubic profile for the moisture-content level. We present the results below without proof:

(i) *Low-intensity drying*, $\mathscr{N}_0 \leqslant 2$

$$\Phi = f(1 - (1-f)/f\gamma Bi)^2 . \quad (65)$$

(ii) *High-intensity drying*, $\mathscr{N}_0 > 2$

$$\Phi = \frac{1 - (1-f)/f\gamma Bi - 3/4\mathscr{N}_0 f}{1 - 3/4\mathscr{N}_0} . \quad (66)$$

In the above expressions, the intensity of drying $\mathscr{N}_0$ is defined in terms of the limiting diffusion coefficient $\mathscr{D}_{a|X \to 0}$.

Equation (65) for low-intensity drying is similar to that for the case of constant diffusivity [equation (44)], while equation (66) for high-intensity drying differs from that for the case of constant diffusivity only in the coefficient of the terms in $\mathcal{N}$.

DISCUSSION

The characteristic drying curves (f vs Φ) for a constant-diffusivity process are plotted in Fig. 7 for $\gamma Bi = 1, 2, 5$ and 10 and for various contours of $\mathcal{N}$. As γBi increases, so the contour for low-intensity drying

In a given dryer, the variations of total solids temperature and external mass-transfer coefficient and the influence of the small changes in γBi is second-order. Under these conditions, the concept of a single characteristic drying curve will be good enough for interpolating conditions in a continuous dryer, as the material is subjected to the same relative intensity $\mathcal{N}$ at the solids inlet. In a batch dryer, $\mathcal{N}$ varies with position, exponentially falling with distance from the air inlet:

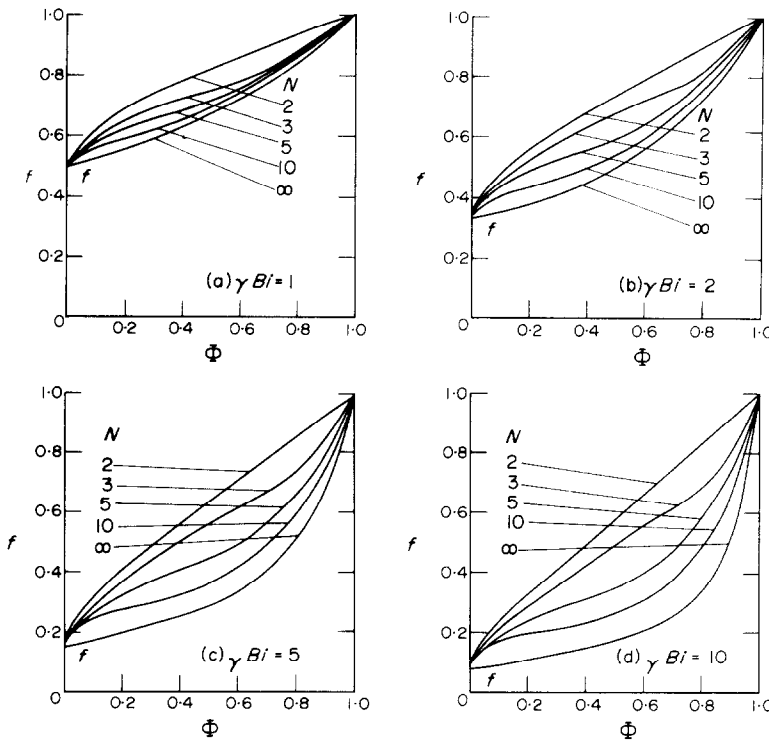$$\mathcal{N} = \mathcal{N}_{in} \exp(-\zeta) \tag{67}$$

FIG. 7. Characteristic drying curves for a constant-diffusivity process: (a) $\gamma Bi = 1$: (b) $\gamma Bi = 2$: (c) $\gamma Bi = 5$: (d) $\gamma Bi = 10$.

becomes more linear and the contours for high-intensity drying are more markedly concave upwards. The curves illustrate that there is no single characteristic curve for a given material. The range of dependent variables of f for a given material are listed below:

Dependent variables for given material	
$\mathcal{N} \leqslant 2$	γ (temperature), Bi (material thickness, external mass-transfer coefficient), Φ (mean moisture content)
$\mathcal{N} > 2$	γ (temperature), Bi (material thickness, external mass-transfer coefficient), $\mathcal{N}$ (relative intensity), Φ (mean moisture content)

where ζ is the number of transfer units to the considered position. Should $\mathcal{N}$ be greater than the critical value of 2 over part or whole of the dryer, then some allowance for the variation of $\mathcal{N}$ should be made. One can operate, for instance, a calculational procedure based on drying curve for one value of which is adjusted progressively in terms of the actual local value of $\mathcal{N}$ according to expressions such as equations (56) and (66). An allied calculational procedure is the continual redefinition of the characteristic moisture content in terms of a variable critical moisture content based upon an experimental (or theoretical) critical-point curve.

When one dries a thick material slowly ($\mathcal{N} < 2$, $\gamma Bi \gg 1$), the characteristic drying curve approaches the

straight line $f = \Phi$. This conclusion appears to be corroborated by drying data for $100 \times 25\,mm$ Pinus radiata boards [12].

REFERENCES

1. G. Nonhebel and A. A. H. Moss, *Drying of Solids in the Chemical Industry*, p. 52. Butterworths, London (1971).
2. R. B. Keey, *Drying Principles and Practice*, pp. 210–269. Pergamon Press, Oxford (1972).
3. R. E. Treybal, *Mass-Transfer Operations*, 2nd Edn, p. 40. McGraw-Hill, New York (1968).
4. C. H. Bedingfield, Jr. and T. B. Drew, Analogy between heat transfer and mass transfer, *Ind. Engng Chem.* **42**, 1164 (1950).
5. J. C. Ashworth and R. B. Keey, The evaporation of moisture from wet surfaces, *Chem. Engng Sci.* **27**, 1797 (1972).
6. M. Suzuki and S. Maeda, On the mechanism of drying of granular beds, *J. Chem. Engng Japan* **1**, 26 (1968).
7. O. Krischer, *Die wissenschaftlichen Grundlagen der Trocknungstechnik*, Vol. 1, 2nd Edn, pp. 298–306. Springer, Berlin (1963).
8. R. E. Peck and J. Y. Kauh, Evaluation of drying schedules, *A.I.Ch.E. Jl* **15**, 85 (1969).
9. R. P. Morgan and S. Yerazunis, Heat and mass transfer during liquid evaporation from porous materials, *Chem. Engng Prog. Symp. Ser.* No. 79, **63**, 1 (1967).
10. A. H. Nissan, W. G. Kaye and J. R. Bell, Mechanism of drying thick porous bodies during the falling rate period—1. The pseudo-wet-bulb temperature, *A.I.Ch.E. Jl* **5**, 103 (1959).
11. R. Toei, S. Hayashi, J. Hasegawa, T. Kai and S. Tanaka, Mechanism of drying of a bed of granular and powdered materials during the second falling rate period, *Kagaku Kōgaku* **28**, 458 (1964).
12. R. B. Keey, Simulation of the kiln-seasoning of timber, Proc. 4th. Internat. Congress Chisa, Section D5 Capillary Structures, Praha, (1972).
13. J. W. Sutherland, P. J. Banks and H. J. Griffiths, Equilibrium heat and moisture transfer in air flow through grain, *J. Agric. Engng Res.* **16**, 368 (1971).
14. M. Suzuki and S. Maeda, Mechanism of drying non-hygroscopic materials, Proc. III Japan Heat Transfer Symposium, p. 161, Sendai (1966).

APPENDIX

The Penetration of the Drying Front in High-Intensity Drying

We shall assume that the movement of moisture is Fickian, so that over the zone from the surface to the full depth of penetration of the drying front Fick's second law describes the change in moisture content. This may be written in the form

$$\frac{\partial X}{\partial \tau} = \mathcal{D}_a \frac{\partial^2 X}{\partial z^2}. \tag{A1}$$

For ease of book-keeping we shall introduce the dimensionless ratios:

θ—relative time, $\theta = \tau/\tau_1$
η—relative distance, $\eta = z/b$.

The time τ_1 is the time of drying when the front has reached the base of the material ($\delta = 1$), and b is the thickness of the

material itself. With these ratios equation (A1) becomes

$$\frac{\partial X}{\partial \theta} = \left[\frac{\tau_1 \mathcal{D}_a}{b^2} \right] \frac{\partial^2 X}{\partial \eta^2}. \tag{A2}$$

The coefficient $[\tau_1 \mathcal{D}_a/b^2]$ is the Fourier number for the penetration depth to extend throughout the material, Fo_1.

Equation (A2), on integrating between 0 and δ with respect to η, becomes

$$\frac{d}{d\theta} \left\{ \int_0^\delta X \, d\eta - X_0 \delta \right\} = Fo_1 \left\{ \frac{\partial X}{\partial \eta} \bigg|_{\eta=\delta} - \frac{\partial X}{\partial \eta} \bigg|_{\eta=0} \right\}. \tag{A3}$$

We shall fit a parabolic equation of the form

$$X = a_0 + a_1 \eta + a_2 \eta^2 \tag{A4}$$

to equation (A3). The unknown coefficients can be found from the boundary conditions:

$$X|_{\eta=\delta} = X_0 \tag{b.c.1}$$

$$\frac{\partial X}{\partial \eta} \bigg|_{\eta=\delta} = 0 \tag{b.c.2}$$

$$\frac{\partial X}{\partial \eta} \bigg|_{\eta=0} = \frac{N_A^0 b}{\rho_s \mathcal{D}_a}. \tag{b.c.3}$$

These boundary conditions lead to the moisture-content profile

$$X = X_0 - \frac{N_A^0 b}{2\rho_s \mathcal{D}_a \delta} [\delta - \eta]^2. \tag{A5}$$

Substitution of this profile into equation (A3) leads to the expression

$$-\frac{N_A^0 b}{6\rho_s \mathcal{D}_a} \cdot \frac{d}{d\theta} [\delta^2] = 0 - Fo_1 \cdot \frac{N_A^0 b}{\rho_s \mathcal{D}_a} \tag{A6}$$

which simplifies to

$$\frac{d}{d\theta} [\delta^2] = 6Fo_1. \tag{A7}$$

Equation (A7) has the solution:

$$\delta = \sqrt{(6Fo_1 \theta)}. \tag{A8}$$

Equation (A8) may be used to eliminate δ from the moisture-content profile [equation (A5)]:

$$X = X_0 - \frac{N_A^0 b}{2\rho_s \mathcal{D}_a (6Fo \cdot \theta)^{1/2}} [(6Fo \cdot \theta)^{1/2} - \eta]^2. \tag{A9}$$

At the exposed surface, $\eta = 0$: equation (A9) then yields the expression

$$X|_{\eta=0} = X_0 - \frac{3}{2} \left[\frac{N_A^{02} \tau_1}{\rho_s^2 \mathcal{D}_a} \right]^{1/2} \tag{A10}$$

for the surface-moisture content after some manipulation. The time taken for this moisture content to be reduced to zero is thus:

$$\tau_c = \frac{2}{3} \mathcal{D}_a (\rho_s X_0 / N_A^0)^2. \tag{A11}$$

Or, in terms of relative time θ_s the length of the first drying period b becomes

$$\theta_c = \frac{2}{3} \frac{\mathcal{D}_a}{\tau_1} \left[\frac{\rho_s X_0}{N_A^0} \right]^2 \tag{A12}$$

However, if the high-intensity drying pertains, the drying front must still not have penetrated the whole body. The depth of penetration, then, is subject to the restraint

$$\delta(\theta_c) = \sqrt{(6Fo_1\theta_c)} \leqslant 1. \tag{A13}$$

On substituting the value of θ_c from equation (A12) and noting that $Fo_1 = \tau_{1a}/b^2$, we find

$$\delta = 2\rho_s X_{0a}/N_A^0 b \leqslant 1 \tag{A14}$$

or

$$N_A^0 b/\rho_s X_0 D_a > 2. \tag{A15}$$

The parameter $N_A b/\rho_s X_0 \mathscr{D}_a$ represents the relative intensity of drying $\mathscr{N}$. Thus, when $\mathscr{N} > 2$, a drying front narrower than the body thickness is observed. Whenever $\mathscr{N} < 2$, no such front is seen, and the moisture content at the base of the material, X_1, will have fallen below the initial value at the critical point.

DE LA COURBE CARACTERISTIQUE DE SECHAGE

Résumé—On examine le fondement théorique de la courbe caractéristique de séchage en considérant le séchage d'une tranche poreuse et non-hygroscopique d'étendue infinie. Dans les conditions d'un séchage intensif, lorsqu'une vague de séchage apparait, la courbe caractéristique de séchage est une fonction de l'intensité relative de séchage, mais ce n'est pas le cas sous les conditions de séchage moins intensif. A la limite du séchage lent des matériaux épais et modérément imperméables, la courbe caractéristique de séchage est linéaire.

ÜBER DIE CHARAKTERISTISCHE TROCKNUNGSKURVE

Zusammenfassung—Um die theoretische Begründung des Modells der charakteristischen Trocknungskurve zu überprüfen, wird die Austrocknung einer unendlich ausgedehnten, nichthygroskopischen, porösen Platte betrachtet. Wenn bei intensiven Trocknungsbedingungen eine Trocknungswelle auftritt, ist die charakteristische Trocknungskurve eine Funktion der relativen Trocknungsintensität, während sie dies bei weniger intensiven Trocknungszuständen nicht ist. Eine lineare Trocknungskurve stellt sich lediglich als Grenzkurve bei der langsamen Trocknung von dicken, fast undurchlässigen Stoffen ein.

К ВОПРОСУ О ХАРАКТЕРИСТИЧЕСКОЙ КРИВОЙ СУШКИ

Аннотация — Приводится теоретическое обоснование характеристической крчвой сушки применительно к процессу обезвоживания пористой негигроскопической неограниченной пластины. Показано, что в условиях интенсивной сушки, когда возникает фронт обезвоживания, характеристическая кривая сушки зависит от относительной скорости сушки, в то время как в условиях низкой интенсивности процесса эта закономерность не наблюдается. В предельном случае, при низкоинтенсивной сушке толстых плохо проницаемых материалов, получается единая линейная характеристическая кривая сушки.